Int. J. Heat Mass Transfer. Vol. 22, pp. 763–769
Pergamon Press Ltd. 1979. Printed in Great Britain

GENERALIZATION OF HEAT-TRANSFER RESULTS FOR TURBULENT FREE CONVECTION ADJACENT TO HORIZONTAL SURFACES

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(*Received* 15 *August* 1977 *and in revised form* 7 *April* 1978)

Abstract—The common character of upward heat transfer in bottom-heated and internally heated fluid layers is demonstrated. This is accomplished by comparing their heat-transfer characteristics on the basis of a modified Nusselt number, defined in terms of an implicit length scale, in contrast to the conventional Nusselt number which contains the total layer depth. The implicit length scale is derived from dimensional considerations and depends only upon parameters relevant to the thermal boundary layer adjacent to the solid surface. The modified Nusselt number was found to have an extremely weak dependence upon the Rayleigh number, the variation being only two-fold over a 10^7-fold variation of Rayleigh number. More importantly, the heat transfer data for bottom-heated layers (i.e. Rayleigh–Bénard convection) were shown to be almost identical to those for internally heated layers. These results suggest that factors outside of the boundary layer, such as the method of heating, have little influence upon the heat-transfer coefficients of the two systems. The relationship between the implicit boundary-layer length scale used herein and the critical boundary-layer thickness used in the boundary-layer instability models of Howard and others is discussed. Least square correlation of the combined data for both bottom and internal heating is also presented.

NOMENCLATURE

D, total layer depth;
D^+, far-field length scale;
g, gravitational acceleration;
h, heat-transfer coefficient;
k, thermal conductivity;
l^*, near-field length scale, equation (8);
Nu, Nusselt number;
Nu_0, Nusselt number at lower boundary;
Nu_1, Nusselt number at upper boundary;
Nu^*, modified (boundary-layer) Nusselt number, equation (9);
q, heat flux;
q_0, heat flux at lower boundary;
q_1, heat flux at upper boundary;
Ra, Rayleigh number;
Ra_I, internal Rayleigh number;
Ra^+, far-field Rayleigh number;
Ra_{δ_c}, critical boundary-layer Rayleigh number;
S, volumetric heat generation rate;
T_m, maximum temperature in internally-heated layer;
ΔT, temperature difference, $T_0 - T_1$;
ΔT_0, temperature difference, $T_m - T_0$;
ΔT_1, temperature difference, $T_m - T_1$;
ΔT^+, far-field temperature difference;
ΔT^*, near-field temperature difference.

Greek symbols

β, coefficient of thermal expansion;
δ_c, critical boundary-layer thickness;
κ, heat diffusivity;
ν, kinematic viscosity.

Subscripts

0, lower surface;
1, upper surface;
m, maxima.

Superscripts

*, near-field quantity;
+, far-field quantity.

1. INTRODUCTION

RAYLEIGH–BÉNARD convection in a horizontal fluid layer heated from below is the prototype for a large class of problems concerning fluid flow driven by an unstable buoyancy force distribution. Another member of this class, one which has received considerable attention in recent years, is natural convection in an internally heated fluid layer cooled from above and below. In contrast to the Rayleigh–Bénard case, convection in an internally heated layer is confined to the upper part of the layer within which the temperature distribution is destabilizing, the lower part being relatively stagnant and stably stratified. Nevertheless, the Rayleigh–Bénard problem and the problem of convection induced by internal heating have certain basic features in common, particularly with regard to

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the mechanisms governing the transfer of heat at the upper boundary of each system. For example, if the convection is fully turbulent, the flow in both cases is characterized by a nearly isothermal core together with a thin thermal boundary layer adjacent to the upper surface. In this situation, it is reasonable to expect that the upward heat transfer is controlled mainly by local parameters associated with the thermal boundary layer and that the method of heating is of secondary importance. Consequently, it should be possible to correlate the combined heat-transfer data for bottom-heated and internally heated layers in a manner that reveals the common physical basis of the two turbulent convection processes. Yet, to date, there has been no attempt to draw a quantitative comparison between the heat-transfer results for the two problems. In fact, the heat-transfer data for internally heated layers is usually presented in a form which makes such a comparison difficult. The purpose of this communication is to suggest a simple method of displaying the combined data, a method based upon an important physical characteristic shared by both problems. In so doing, the heat-transfer correlations for the bottom-heated and internally heated layers are shown to be almost identical.

The emphasis throughout this work is on deducing the common mechanisms governing upward heat transfer in bottom-heated and internally heated layers via the existing heat-transfer data for both problems. Thus, the particular correlation employed here, though physically meaningful, is not necessarily the most appropriate for practical calculations. In special circumstances, other correlations might be more useful. In any event, it is felt that the physics of the turbulent convection process can be best understood by the method described below.

2. COMPARISON OF THE SEVERAL RELATED PROBLEMS

The classical Rayleigh–Bénard problem, which has been studied most extensively, concerns a fluid layer heated from below and bounded horizontally by non-slip, isothermal surfaces. It has been customary to present the heat-transfer results in terms of a correlation of Nusselt number vs Rayleigh number, Prandtl number effects being insignificant in the moderate to large Prandtl number range. Both the Nusselt number, Nu, and the Rayleigh number, Ra, are defined on the basis of the full layer depth, D, and the total temperature difference, ΔT, between the horizontal boundaries, as follows;

$$Nu = \frac{qD}{k\Delta T}, \tag{1}$$

$$Ra = \frac{g\beta\Delta T D^3}{\nu\kappa}. \tag{2}$$

Recently, a number of studies of convection in volumetrically heated fluid layers, under various combinations of isothermal and adiabatic boundary conditions, have appeared in the literature. In the internally heated case, the magnitude of the destabilizing temperature difference is a function of the imposed volumetric heating rate. Consequently, investigators have chosen to correlate their data for average heat flux at the boundaries in terms of upward and downward Nusselt numbers, Nu_1 and Nu_0, and a modified internal Rayleigh number, Ra_I, based upon the known strength, S, of the volumetric heat source. The relevant parameters are defined by

$$Nu_1 = \frac{q_1 D}{k\Delta T_1}, \tag{3}$$

$$Nu_0 = \frac{q_0 D}{k\Delta T_0}, \tag{4}$$

$$Ra_I = \left(\frac{g\beta}{\nu\kappa}\right)\left(\frac{SD^2}{2k}\right)D^3. \tag{5}$$

As would be expected, the resulting heat-transfer correlations for the internally heated layer do not bear any simple relationship to those for the bottom-heated (Rayleigh–Bénard) layer.

In the bottom-heated case, at high Rayleigh numbers, it is well known that the Nusselt number is approximately proportional to the 1/3-power of the Rayleigh number, indicating that the length parameter, D, has only a very weak influence upon the heat-transfer characteristics of the fluid layer. This fact, and other observations, has led a number of investigators [1–5] to suggest models of the turbulent convective flow which concentrate upon the two thermal boundary layers at the horizontal surfaces, within which nearly all of the temperature variation is confined. The present study also makes use of the boundary-layer concept; but, instead of proposing a new convection model or lending support to a specific existing one, we employ the boundary-layer-dominant aspect of the turbulent thermal convection problem as well as dimensional reasoning simply as a basis for comparing the various experimental data for bottom-heated and internally heated layers.

In the remainder of this work, we restrict our attention, in the case of internally heated layers, to the region adjacent to the upper boundary, within which the temperature variation is destabilizing and the heat flux is directed upward. The downward heat flux in volumetrically heated layers cooled from below is largely controlled by conduction through a relatively stagnant fluid sublayer near the lower boundary. In that situation, one cannot appeal to the boundary-layer concept in the manner described in this study.

Examinations of the temperature distributions in internally heated layers undergoing turbulent convection show that, just as in the case of Rayleigh–Bénard convection, the temperature variation in the upper portion of the layer is restricted to a very thin boundary layer adjacent to the top surface. This suggests that, in both the internally heated and bottom-heated cases, the heat transfer is likely to be determined by near-field parameters associated with the boundary layer rather than far-field parameters associated with the system as a whole. Hence, for the purpose of comparing experimental data, it is inappropriate to use heat-transfer correlations based upon Nusselt numbers defined by equations (1) or (3), in which the total layer depth, D, and the temperature difference, ΔT, are far-field parameters. Instead, a more appropriate characteristic temperature difference is that across the thermal boundary layer. This new temperature scale, ΔT^*, is defined as

$$\Delta T^* = \tfrac{1}{2}\Delta T \text{ for bottom-heated layers} \qquad (6)$$

and

$$\Delta T^* = T_m - T_1 \text{ for internally heated layers.} \qquad (7)$$

In writing down equation (6), we are ignoring the small core temperature reversals known to occur in laminar regimes at low Rayleigh numbers, which are outside our primary range of interest. Also, the possible existence of small temperature variations in the turbulent core is neglected since they are insignificant in the Rayleigh number ranges explored so far.

Instead of using the far-field parameter, D, in the definition of the Nusselt number, it would be more appropriate to use a length scale depending only on near-field parameters such as ΔT^* and the physical properties. Dimensional considerations show that an implicit length scale, l^*, which satisfies this requirement is

$$l^* = \left(\frac{\nu\kappa}{g\beta\Delta T^*}\right)^{1/3}. \qquad (8)$$

The definitions given above lead to a Nusselt number, Nu^*, defined entirely in terms of near-field parameters:

$$Nu^* = \frac{ql^*}{k\Delta T^*}, \qquad (9)$$

where q is the heat flux across the thermal boundary layer at the upper boundary. If indeed, the average surface heat flux is controlled by the boundary layers, then Nu^* should be only weakly dependent upon the Rayleigh number, which still contains the far-field length scale, D. Also, the heat-transfer correlations for the Rayleigh–Bénard problem and the problem of convection with internal heat generation should be identical, or very nearly so. We now examine the validity of these hypotheses by considering the published heat-transfer data from [6–12].

First, to facilitate a comparison of the experimental data, the Rayleigh numbers for the bottom-heated and internally-heated layers will be transformed to a common basis. A far-field Rayleigh number, Ra^+, will be defined by

$$Ra^+ = \frac{g\beta\Delta T^+ D^{+3}}{\nu\kappa}, \qquad (10)$$

where the far-field temperature difference, ΔT^+, and length scale, D^+, are evaluated according to the following criteria:

(a) *Bottom-heated layers with isothermal boundaries (Rayleigh–Bénard problem)*

In this case, there are two similar boundary layers, one heated from below, the other cooled from above. The postulate that far-field effects are negligible also implies very small interaction between these boundary layers (at least as regards the average heat-transfer behavior). Thus,

$$\Delta T^+ = \tfrac{1}{2}(T_0 - T_1) = \Delta T^*, \qquad (11)$$

$$D^+ = \tfrac{1}{2}D. \qquad (12)$$

(b) *Internally heated layers with isothermal boundaries at the same temperature*

The temperature and length scales for the upper region are

$$\Delta T^+ \doteq T_m - T_1 = \Delta T^*, \qquad (13)$$

$$D^+ = \left(\frac{q_1}{q_0+q_1}\right)D. \qquad (14)$$

The justification for (14) is that only the volume heating occurring in D^+ contributes to the upward heat transfer q_1. Note also that q_1 and q_0 are proportional to Nu_1 and Nu_0, which are given in [9]. Clearly, one must use the measured values for the upward and downward heat fluxes, q_1 and q_0, to obtain a precise magnitude for D^+. This poses no problem in this study, because our primary aim is not to predict heat-transfer rates but rather to establish a rational basis upon which to compare the existing heat-transfer data for the two types of processes.

(c) *Internally heated layers with isothermal upper boundary and adiabatic lower boundary*

In this instance, the maximum temperature occurs at the insulated lower boundary; thus,

$$\Delta T^+ = T_0 - T_1 = \Delta T^*, \qquad (15)$$

$$D^+ = D. \qquad (16)$$

Note that $\Delta T^+ = \Delta T^*$ in all cases under consideration and this is consistent with the physical argument that nearly all temperature variations are confined within the boundary layer. The above definitions appear to be the most reasonable and physically meaningful for our purposes. Of course, it might be possible to invoke qualitative arguments to arrive at a different set of definitions, but it is unlikely that the most appropriate set could be selected on the basis of the experimental data alone.

The experimental data of Kulacki and Goldstein [9] and of Ralph *et al.* [10] for internally heated water layers between isothermal, equal temperature boundaries are now compared with data for Rayleigh–Bénard convection taken from Chu and Goldstein [6], Garon and Goldstein [7], and Threlfall [8]. Earlier data for bottom-heated layers have been reviewed by the second group of investigators and are in substantial agreement with their findings. The comparison of the heat transfer results for the two types of heating are shown in Fig. 1. Since the numerical data from the experiments of Ralph *et al.* [10] were not available for plotting, it was necessary to display their correlated results, representing about 40 data points, as the single solid line. We see

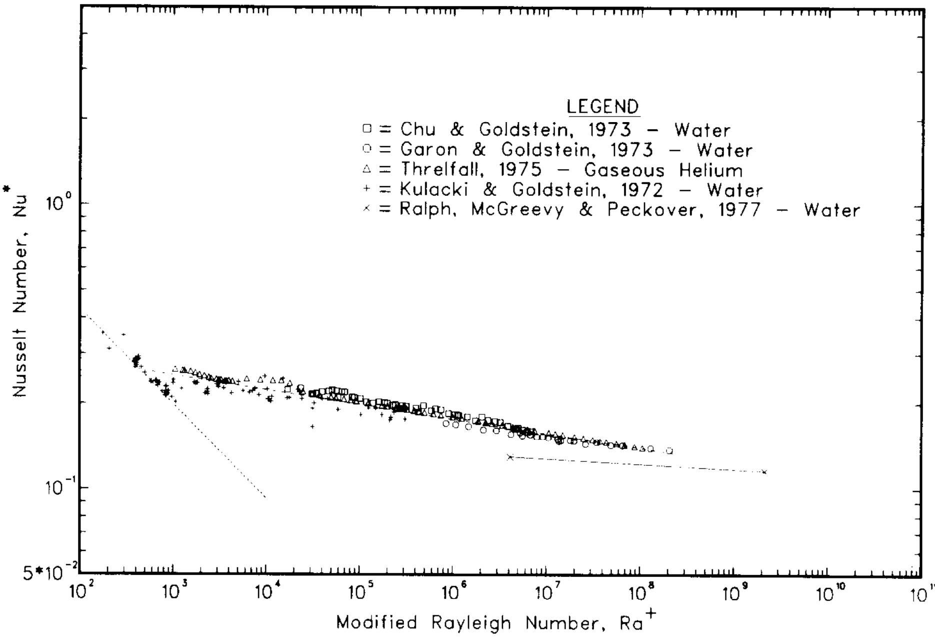

FIG. 1.

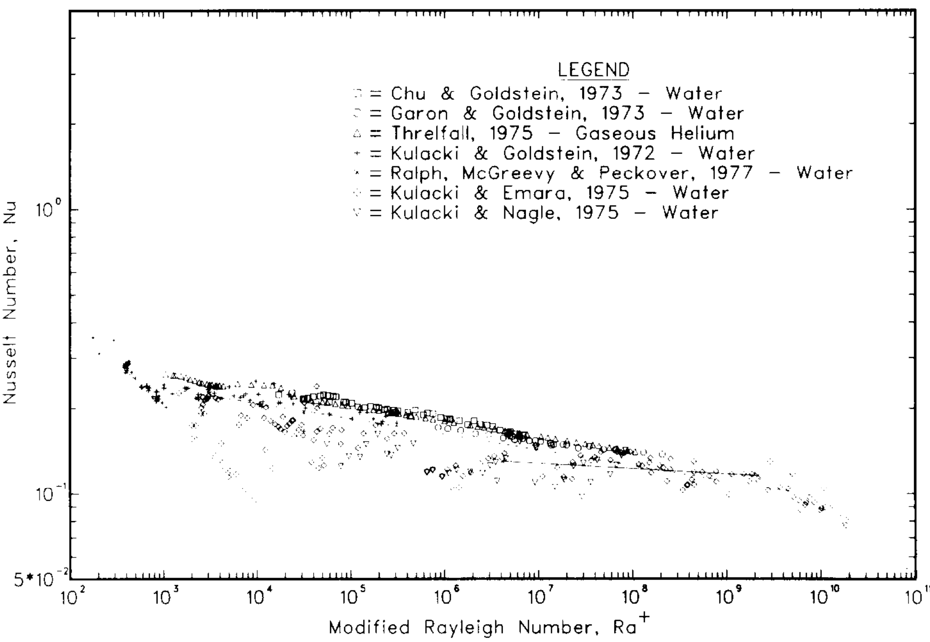

FIG. 2.

that, in the domain $1 \times 10^3 \leqslant Ra^+ \leqslant 3 \times 10^5$, that is, in the turbulent or near turbulent regime, the data from [9] for internal heating coincides quite closely with the data from [6–8] for bottom-heating. The high Rayleigh number data from [10] for internal heating, in the range $4 \times 10^6 \leqslant Ra^+ \leqslant 2 \times 10^9$, also shows very good agreement with the Rayleigh–Bénard data.

The data from Kulacki and Nagle [11] and Kulacki and Emara [12] for internally heated water layers having an insulated lower boundary and a cooled upper boundary is shown in Fig. 2, together with the data from Fig. 1. These last two sets of data show considerably more scatter than that from the other experiments. This is more likely due to experimental uncertainties rather than to any inherent physical process. Even so, the results for the internally heated case with adiabatic lower boundary still are reasonably consistent with those for Rayleigh–Bénard convection and for internal heating with isothermal boundaries. It should be noted that use of Nu^*, rather than Nu_1, actually expands the vertical axis. This results in somewhat more scatter in the data than that found in previous plots of Nu_1 vs Ra_I [11,12].

Statistical analyses of the data displayed in Figs. 1 and 2 also were performed.† A linear, least squares correlation of $\log(Nu^*)$ vs $\log(Ra^+)$, was employed to obtain the unknown constants, A and b, in the power law expression

$$Nu^* = A(Ra^+)^{-b}. \quad (17)$$

The results are given in Table 1. Clearly, the various expressions do not differ substantially among themselves. As expected, Nu^* is a weak function of Ra^+. The maximum variation of all of the data from a mean value of $Nu^* \simeq 0.15$ is only of the order of $\pm 50\%$ over the entire range $1 \times 10^3 \leqslant Ra^+ \leqslant 2 \times 10^{10}$.

Thus, the characteristic length scale is directly proportional to the time-averaged boundary-layer thickness. For any given temperature variation within the boundary layer, δ_c is clearly proportional to k/h, implying that

$$\frac{h\delta_c}{k} = Nu^*(Ra_{\delta_c})^{1/3} = \text{constant}. \quad (20)$$

The fact that Nu^* is nearly constant lends some support to the validity of the boundary layer models. On the other hand, the slight dependence of Nu^* on Ra^+ (see Figs. 1 and 2) may be interpreted as an indication of the fact that the Rayleigh numbers so far explored are still too small for these models to be strictly valid, as pointed out by Long [15]. Most recently, Cheung [5] derived a simple boundary layer

Table 1. Correlation equations for various data combinations

Data combinations	Correlation	Range of Ra^+
Rayleigh–Bénard only [6–8]	$Nu^* = 0.395Ra^{+(-0.057)}$	1×10^3–1×10^8
Rayleigh–Bénard plus internal-heating with isothermal boundaries [6–8], [9]	$Nu^* = 0.358Ra^{+(-0.051)}$	1×10^3–2×10^8
Rayleigh–Bénard plus all internal-heating [6–8], [9], [11–12]	$Nu^* = 0.346Ra^{+(-0.055)}$	1×10^3–2×10^{10}
Ralph *et al.* [10]—internal heating with isothermal boundaries	$Nu^* = 0.173Ra^{+(-0.019)}$	4×10^6–2×10^9

3. RELATIONSHIP TO BOUNDARY-LAYER INSTABILITY MODELS

Howard [1] proposed that the average thickness of the thermal boundary layer is governed by a repetitive process of boundary-layer growth and draining by departing thermals. His model was later modified and amplified by a number of other workers [2–5]. Basically, these models postulate that there exists a critical Rayleigh number for instability of the thermal boundary layer such that the mean layer thickness attains a critical value for a given ΔT^*. The critical Rayleigh number is defined as

$$Ra_{\delta_c} = \frac{g\beta\Delta T^*\delta_c^3}{\nu\kappa}, \quad (18)$$

where δ_c is the critical layer thickness. Assuming conduction to be the dominant mode of heat transfer within δ_c, the familiar 1/3-power dependency of Nu on Ra follows. Note that the exact value of Ra_{δ_c} depends on the choice of definition for the boundary-layer thickness. Since at the current stage of development, most of these models are only semi-quantitative, we shall not concern ourselves with published estimates of Ra_{δ_c} [13,14]. Substituting equation (8) into (18), we obtain

$$l^* = (Ra_{\delta_c})^{-1/3}\delta_c. \quad (19)$$

†The results from [10] were not included in the analyses since the numerical data points were not available.

equation based on the measured heat transfer data for Rayleigh–Bénard layers and internally heated layers with adiabatic lower boundary. For the range of Rayleigh numbers studied, the critical Rayleigh number Ra_{δ_c} was shown to be a function of δ_c/D.

Finally, it should be emphasized that l^* has been derived purely from dimensional considerations. The usefulness of the Nu^* correlation for combining the heat-transfer data for the bottom-heated and internally heated layers clearly indicates that this choice of a length scale for the definition of Nu^* is physically meaningful. However, whether the physics of the model is related to the postulated thermal boundary-layer instability phenomenon cannot be decided from considerations of the heat-transfer data alone.

4. CONCLUSIONS

The surface heat-transfer coefficient in turbulent convection in horizontal layers depends primarily upon the near-field parameters, regardless of the method of heating. Consequently, in the turbulent convective regime, the heat-transfer characteristics of Rayleigh–Bénard convection and convection with internal heat generation can be derived one from the other. This is accomplished by defining a boundary-layer Nusselt number, Nu^*, based upon the diffusive length and temperature scales, l^* and ΔT^*, which naturally characterize the thermal boundary layer. The resulting power law correlation of Nu^* vs Ra^+

was found to be quite effective in combining the heat-transfer data to show the fundamental similarity between the two convection processes which formerly were treated as separate entities.

Acknowledgements—The authors wish to thank Ms. Marilyn Goldman for typing the manuscript. The work at Argonne was performed under the auspices of the U.S. Department of Energy. Part of the work by the second author was performed at the University of Illinois under NSF grant ENG74-23084.

REFERENCES

1. L. N. Howard, Convection at high Rayleigh number, in *Proceedings of the 11th International Congress on Applied Mechanics*, pp. 1109–1115. Springer, Berlin (1966).
2. M. M. Chen, A model for turbulent thermal convection in horizontal fluid layers, *Bull. Am. Phys. Soc.* **16**, 1310 (1971).
3. K. G. T. Hollands, G. D. Raithby and L. Konicek, Correlation equations for free convection heat transfer in horizontal layers of air and water, *Int. J. Heat Mass Transfer* **18**, 879–884 (1975).
4. M. J. Manton, On the high Rayleigh number heating of a fluid above a horizontal surface, *Appl. Sci. Res.* **31**, 267–277 (1975).
5. F. B. Cheung, Correlation equations for turbulent thermal convection in a horizontal fluid layer heated internally and from below, *J. Heat Transfer* **100**, 416–422 (1978).
6. T. Y. Chu and R. J. Goldstein, Turbulent convection in a horizontal layer of water, *J. Fluid Mech.* **60**, 141–159 (1973).
7. A. M. Garon and R. J. Goldstein, Velocity and heat transfer measurements in thermal convection, *Physics Fluids* **16**, 1818–1825 (1973).
8. D. C. Thelfall, Free convection in low-temperature gaseous helium, *J. Fluid Mech.* **67**, 17–28 (1975).
9. F. A. Kulacki and R. J. Goldstein, Thermal convection in a horizontal fluid layer with uniform volumetric energy sources, *J. Fluid Mech.* **55**, 217–287 (1972).
10. J. C. Ralph, R. McGreevy and R. S. Peckover, Experiments in turbulent thermal convection driven by internal heat sources, in *Heat Transfer and Turbulent Buoyant Convection*, Vol. II, pp. 587–599. Hemisphere, Washington (1977).
11. F. A. Kulacki and M. Z. Nagle, Natural convection in a horizontal fluid layer with volumetric energy sources, *J. Heat Transfer* **97**, 204–211 (1975).
12. F. A. Kulacki and A. A. Emara, High Rayleigh number convection in enclosed fluid layers with internal heat sources, Tech. Report 3952-1, NUREG-75/065, NRC-3 (1975).
13. E. M. Sparrow, R. B. Husar and R. J. Goldstein, Observations and other characteristics of thermals, *J. Fluid Mech.* **41**, 793–800 (1970).
14. F. Davenport and C. J. King, A note on Howard's model for turbulent natural convection, *J. Heat Transfer* **97**, 476–478 (1975).
15. R. R. Long, Relation between Nusselt number and Rayleigh number in turbulent thermal convection, *J. Fluid Mech.* **73**, 445–451 (1976).

GENERALISATION DES RESULTATS POUR LA CONVECTION THERMIQUE NATURELLE, TURBULENTE ET ADJACENTE A DES SURFACES HORIZONTALES

Résumé—On montre le caractère commun du transfert thermique ascendant dans les couches fluides chauffées à leur base ou à coeur. Ceci est obtenu en comparant les caractéristiques de transfert thermique à partir d'un nombre de Nusselt modifié, défini avec une échelle de longueur implicite, contrairement au nombre de Nusselt conventionnel qui contient la hauteur de la couche. L'échelle de longueur implicite est obtenue par des considérations dimensionnelles et elle dépend seulement des paramètres relatifs à la couche limite thermique adjacente à la surface solide. Le nombre de Nusselt modifié varie faiblement en fonction du nombre de Rayleigh, soit un doublement pour le produit par 10^7 du nombre de Rayleigh. Il est important de noter que le transfert thermique pour les couches chauffées à leur base (par exemple, convection de Rayleigh–Benard) est à peu près identique à celui des couches chauffées à coeur. Ceci suggère que les facteurs extérieurs à la couche limite, tels que la méthode de chauffage, ont une faible influence sur les coefficients de transfert thermique des deux systèmes. On discute la relation entre l'échelle de longueur implicite utilisée dans les modèles d'instabilité de couche limite par Howard et autres. On présente aussi des formules de moindre carré des résultats combinés pour les chauffages à la base et interne.

VERALLGEMEINERUNG VON WÄRMEÜBERTRAGUNGSERGEBNISSEN FÜR TURBULENTE FREIE KONVEKTION NAHE HORIZONTALEN OBERFLÄCHEN

Zusammenfassung—Die Gleichartigkeit der nach oben gerichteten Wärmeübertragung für Fluid-Schichten, die vom Boden her bzw. im Inneren beheizt werden, wird gezeigt. Dies wird durch Vergleich ihrer Wärmeübertragungseigenschaften auf der Basis einer modifizierten Nusselt–Zahl erreicht, welche mit Hilfe eines impliziten Längenmaßstabs definiert ist im Gegensatz zur üblichen Nusselt–Zahl, die mit der gesamten Schichtdicke gebildet wird. Der implizierte Längenmaßstab wurde nach Dimensionsbetrachtungen abgeleitet und hängt nur von Parametern ab, die für die thermische Grenzschicht nahe der festen Oberfläche von Bedeutung sind. Es wurde festgestellt, daß die mofifizierte Nusselt–Zahl nur eine extrem schwache Abhängigkeit von der Rayleigh–Zahl hat und sich bei einer Veränderung der Rayleigh–Zahl um den Faktor 10^7 nur um den Faktor 2 ändert. Noch wichtiger: es wurde gezeigt, daß die Wärmeübertragungswerte für vom Boden her beheizte Schichten (d.h., Rayleigh–Bénard Konvektion) beinahe identisch sind mit denen für im Innern beheizte Schichten. Diese Ergebnisse deuten darauf hin, daß Faktoren außerhalb der Grenzschicht, wie die Methode der Beheizung, wenig Einfluß auf die Wärmeübergangskoeffizienten der beiden Systeme haben. Die Beziehung zwischen dem hier benutzten impliziten Längenmaßstab für die Grenzschicht und der kritischen Grenzschichtdicke, welche in Grenzschicht-Instabilitätsmodellen von Howard und anderen benutzt wird, diskutiert. Eine Korrelation nach der Methode der kleinsten Quadrate der kombinierten Daten für Beheizung von der Unterseite bzw. im Innern wird ebenfalls angegeben.

ОБОБЩЕНИЕ РЕЗУЛЬТАТОВ ИССЛЕДОВАНИЯ ТЕПЛООБМЕНА ПРИ ТУРБУЛЕНТНОЙ СВОБОДНОЙ КОНВЕКЦИИ У ГОРИЗОНТАЛЬНЫХ ПОВЕРХНОСТЕЙ

Аннотация — Показан общий характер направленного вверх переноса тепла в слоях жидкости, нагреваемых снизу и изнутри. Это оказалось возможным благодаря сравнению характеристик теплообмена слоя на основе модифицированного числа Нуссельта, выраженного через неявный масштаб длины, в отличие от обычного числа Нуссельта, которое включает общую толщину слоя. Неявный масштаб длины выведен из соображений размерности и зависит только от параметров теплового пограничного слоя на поверхности тела. Найдено, что модифицированное число Нуссельта очень слабо зависит от числа Релея, изменяясь только в два раза при 10^7-кратном изменении числа Релея. Показано, что данные по теплообмену для нагреваемых снизу слоёв (т. е. конвекция Релея-Бенара) почти идентичны данным для слоёв, нагреваемых изнутри. Это говорит о том, что такие факторы, действующие за пределами пограничного слоя, как например способ подвода тепла, оказывают незначительное влияние на коэффициенты теплообмена двух систем. Проведен анализ соотношения между используемым в настоящей работе неявным масштабом длины пограничного слоя и критической толщиной пограничного слоя в моделях неустойчивости пограничных слоёв Ховарда и других. Представлена обобщенная зависимость, полученная путём обработки методом наименьших квадратов данных для случаев подвода тепла как снизу, так и изнутри.